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NONLINEAR ELECTRODYNAMICS AND THE SURFACE REDSHIFT OF PULSARS

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ABSTRACT

Currently is argued that the best method of determining the neutron star (NS) fundamental properties is by measuring the *gravitational redshift* (z) of spectral lines produced in the star photosphere. Measurement of z at the star surface provides a unique insight on the NS mass-to-radius relation and thus on its equation of state (EoS), which reflects the physics of the strong interaction between particles making up the star. Evidence for such a measurement has been provided quite recently by Cottam, Paerels & Mendez (2002), and also by Sanwal et al. (2002). Here we argue that although the quoted observations are undisputed for canonical pulsars, they could be misidentified if the NS is endowed with a super strong B as in the so-called magnetars (Duncan & Thompson 1992) and strange quark magnetars (Zhang 2002), as in the spectral line discovered by Ibrahim et al. (2002, 2003). The source of this new "confusion" redshift is related to nonlinear electrodynamics (NLED) effects.

Subject headings: Gravitation: redshift — line: formation — line: identification — stars: magnetic fields — nonlinear methods: electrodynamics — stars: pulsars: general

1. INTRODUCTION

Neutron stars (NSs), the death throes of massive stars, are among the most exotic objects in the universe. They are supposed to be composed of essentially neutrons, although some protons and electrons are also required in order to guarantee stability against Pauli's exclusion principle for fermions. In view of its density, a neutron star is also believed to trap in its core a substantial part of even more exotic states of matter (EoS). It is almost a concensus that these new states might exist inside and may dominate the star structural properties. Pions plus kaons Bose-Einstein condensates could appear, as well as "bags" of strange quark matter (Miller 2002). This last one believed to be the most stable state of nuclear matter (Glendenning 1997), which implies an extremely dense medium whose physics is currently under severe scrutiny. The major effect of these exotic constituents is manifested through the NS mass-radius ratio (M/R). Most researchers in the field think of the presence of such exotic components not only as to make the star more compact, i.e., smaller in radius, but also to lower the maximum mass it can retain. To get some insight into the neutron star most elusive properties: its mass (M) and radius (R), astronomers use several techniques at disposal, being the most prospective one the *gravitational redshift*. Since the redshift depends on the ratio M/R, then measuring NS spectral lines displacement leads to a direct insight into this dense matter equation of state.

In the late years strong evidence seems to have gathered around a new and exotic class of hyper magnetized neutron stars: the so-called "magnetars" (Duncan & Thompson 1992). These objects are supposed to be the final stage of newly-born neutron stars in which a classical alpha-omega dynamo mechanism has efficiently acted on during the early stages of its evolution, reaching field strentghs up to $B_{\rm Sup-Crit} \sim 10^{17}$ G. A peculiar class of gamma-ray sources known as "soft gamma-ray repeaters" (SGRs) (Kouveliotou et al. 1998), and a set of X-ray pulsars known as of "anomalous" (AXPs), have been claimed to be associated with these type of stars (Mereghetti 1999).

Although these magnetars are said to be the best model to explain the dynamics of SGRs and AXPs, accretion-driven models (Marsden et al. 2001), strange quark matter stars with normal Bs (Zhang, Xu & Qiao 2000; Xu & Busse 2001; Hu & Xu 2002; Xu 2002), or even the strange magnetar interpretation (Zhang 2002) have also been proposed as competing scenarios. Note in passing that in a recent paper Pérez Martínez et al. (2003) have provided arguments contending the formation itself of the so-called magnetars, in the context of the physics used by their mentors (Duncan & Thompson 1992) for proposing their occurrence in nature. Pérez Martínez et al. (2003) argue that a fully description of the physics taking place during the early evolution of NSs should not overlook fundamental issues, as for instance quantum electrodynamics effects, when discussing the rôle of superstrong Bs on the stability of just-born NS pulsars. The *positive magnetization* of the neutron matter and the *appearance of a ferromagnetic configuration* in the star structure are examples of such effects. Thus the idea of magnetars is still contentious. Despite of that lively dispute, in this *Letter*, we present theoretical arguments which alert on the potential effects of NLED in the physics of strongly magnetized NSs.

A very interesting example of how this issue could be elusive is provided by the recent discovery by Ibrahim et al. (2002), and its subsequent confirmation by Ibrahim et al. (2003), of cyclotron resonance features from the SGR 1806-20. It is well-known, in accretion-powered models, that proton (p) and α-particles (He) produce, respectively, fundamental resonances of energy

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$$E_p =^{6.3|^p}_{3.2|_{He}} (1+z)^{-1} \left[\frac{B_{\rm Sup-Crit}}{10^{15}{\rm G}}\right] \text{ keV} . \quad (1)$$

Ibrahim et al. (2002;2003) showed that the 5 keV absorption line in the spectrum of SGR 1806-20 is consistent with a proton-cyclotron fundamental resonance in a redshift-dependent super critical magnetic field (B) of strength: $B_{\rm Sup-Crit} \sim 7.9 \times 10^{14}(1+z)^{-1}$ G. This translates into a $B_{\rm Sup-Crit} \sim 1.0 \times 10^{15}$ G, for the mass and radius of a *canonical* NS ($\rho \sim 10^{14}$ g cm^{-3}, $R \sim 10$ km, $M \sim 1.4\ M_\odot$, $B \sim 10^{12}$ G). An estimate that agrees with the field strength inferred from the SGR 1806-20 spindown, i.e. from P and $\dot{P}$ (Kouveliotou et al. 1998).

2. GRAVITATIONAL VS. NLED REDSHIFT

In particular, we argue that for extremely supercritical magnetic fields NLED effects force photons to propagate along accelerated curves. In case the nonlinear Lagrangean density is a function only of the scalar $F = F_{\mu\nu}F^{\mu\nu}$, to say an approximate Lagrangean

$$L(F) = -\frac{1}{4}F + \frac{\mu}{4}\left(F^2 + \frac{7}{4}G^2\right) , \quad (2)$$

where $\mu = \frac{2\alpha^2}{45}\frac{(\hbar/mc)^3}{mc^2}$, with $\alpha = \frac{e^2}{4\pi\hbar c}$, and $G \equiv F_{\mu\nu}F^{*\,\mu\nu}$; with $F^{*\,\mu\nu} \equiv \frac{1}{2}\eta^{\mu\nu\alpha\beta}F_{\alpha\beta}$, the force accelerating the photons is given by[4]

$$k_{\alpha||\nu}k^\nu = \left(4\frac{L_{FF}}{L_F}F^\mu_\beta F^{\beta\nu}k_\mu k_\nu\right)_{|\alpha} , \quad (3)$$

where k_ν is the wavevector, and L_F means partial derivative with respect to F (note that it does not depend on any intrinsic property of the photons). This feature allows for this force, acting along the photons path, to be geometrized (Novello et al. 2000; Novello & Salim 2001) in such a way that in an effective metric:

$$g^{\rm eff}_{\mu\nu} = g_{\mu\nu} + g^{\rm NLED}_{\mu\nu} , \quad (4)$$

the photons follows geodesic paths, as we shall show in section (III) in the particular case of the Lagrangean called for above. The standard geometric procedure used in general relativity (GR) to describe the photons can now be used upon substituting the usual metric by the effective metric. In particular, the outcoming redshifts prove to have now a couple of components, one due to the gravitational field and another stemming from the B.

A direct insight into the GR $z = z(M,R)$ at the surface of a compact star could be attained from the identification of absorption or emission lines from it. NS mass (M) can be estimated, in some cases, from the orbital dynamics of binary systems, while attempts to measure its radius (R) proceed via high-resolution spectroscopy (Sanwal et al. 2002; upon studying the star 1E1207.4-5209; Cottam, Paerels & Mendez 2002; by analysing type-I X-ray bursts from the star EXO0748-676). In these systems success was achieved in determining these parameters, or the relation in between, by looking at excited ions near the NS surface (arguments favoring a strange star in EXO0748-676 are given by Xu 2003). Gravity effects cause the observed energies of the spectral lines of excited atoms to be shifted to lower values by a factor

$$\frac{1}{(1+z)} \equiv \left(1 - \frac{2G}{c^2}\left[\frac{M}{R}\right]\right)^{1/2} . \quad (5)$$

Measurements of such line properties: energy, width, and polarization; as here called for, would lead to an indirect, but highly accurate estimate of the NS mass-to-radius ratio (M/R); and a tight constraint on its EoS, and to strong limits on the B strength (but not on its configuration) at the star surface. The above analysis stands on whenever effects of NS Bs are negligible. However, if the NS is pervaded by a super strong B ($B_{\rm Sup-Crit}$), then NLED should be taken into account to describe the overall physics taking place on the pulsar surface. Our major result proves that for extremely high Bs the redshift induced by NLED can be as high as the produced by gravity alone, thus making hard to draw any conclusive claim on those NS fundamental properties.

As claimed here, the shift in energy, and width, produced by the effective metric "pull" of the star on laboratory known spectral lines, scales up directly with the strength of the effective potential associated to the effective metric. Thence, this shift has two contributions: one coming from gravitational and another from NLED. For hyper magnetized stars, e.g. magnetars, and if the near surface multipole field is much stronger than the dipole component (see Duncan 1998 for a possible toroidal configuration in SGR 0526-66 based on global seismic oscillations; Section IV discusses implications for the cyclotron line interpretation), the correction factor from NLED is substantial being both contributions of about the same order of magnitude. Thus, there is the possibility, for a given field strength, for gravity effects to be mimicked by electromagnetic (EM) ones, and for the phenomenon to entangle the fixing of constraints on the M/R ratio. This difficulty, we suggest, can be overcome by taking into account that the contribution of the B, that differs from that of the gravitational field which is isotropic, depends on the polarization b^α of the emitted photon, being different for the cases $B_\alpha b^\alpha = 0$ and $B_\alpha b^\alpha \neq 0$.

Our warning is, therefore, that the identification and analysis of spectral lines from high B NSs (in outbursts) must take into account the two possible different polarizations of the received photons, in order to discriminate between redshifts produced either gravitationally or electromagnetially. Putting this result in perspective, we claim that if the characteristic z, or M/R ratio, were to be inferred from this type of sources care should be taken since for this superstrong B such z becomes of the order of the gravitational one expected from a canonical NS. It is, therefore, not clear whether one can cathegorically assert something about the, e.g. SGR 1806-20, M/R ratio under such dynamical conditions. We prove this claim next.

[4] This Lagrangean is built up on the first two terms, because $G = 0$ for a canonical pulsar, of the infinite series expansion associated with the Euler-Heisenberg Lagrangean, which proved to be valid for magnetic field strengths near the quantum electrodynamics critical field $B \sim 10^{13.5}$ G.

3. THE MODEL

The propagation of photons in NLED has been examined by several authors (Bialynicka-Birula & Bialynicki-Birula 1970; Garcia & Plebanski 1989; Dittrich & Gies 1998; De Lorenci, Klippert, Novello & Salim 2000). In the case of geometric optics, where the photon propagation can be identified with the propagation of discontinuities of the EM field in a nonlinear regime, a remarkable property appears: the discontinuities propagate along null geodesics of an effective geometry which depends on the EM field of the background (Novello et al. 2000; Novello & Salim 2001). According to quantum electrodynamics, in Heisenberg & Euler (1936) approximation (see also Schwinger 1951), a vacuum has nonlinear properties, and these novel properties of photon propagation in NLED can show up, in principle, in photons propagating in a vacuum. In this specific case, the equations for the EM field in a vacuum coincide in their form with the equations of continua in which the electric and magnetic permittivity tensors $\epsilon_{\alpha\beta}$ and $\mu_{\alpha\beta}$ are functions of the electric and magnetic fields, determined by some observer represented by its velocity 4-vector V^μ (Denisov, Denisova & Svertilov 2001a, 2001b; Denisov & Svertilov 2003). We should remark that this first order approximation is valid for magnetic fields smaller than B_q, a parameter that will be defined bellow. In curved spacetime, these equations are written as

$$D^\alpha_{||\alpha} = 0, \qquad B^\alpha_{||\alpha} = 0 \;, \tag{6}$$

$$D^\alpha_{||\beta}\frac{V^\beta}{c} + \eta^{\alpha\beta\rho\sigma} V_\rho H_{\sigma||\beta} = 0, \tag{7}$$

$$B^\alpha_{||\beta}\frac{V^\beta}{c} - \eta^{\alpha\beta\rho\sigma} V_\rho E_{\sigma||\beta} = 0, \tag{8}$$

where the double vertical bars "||" stand for covariant derivative, and $\eta^{\alpha\beta\rho\sigma}$ is the completely antisymmetric Levi-Civita tensor. The 4-vectors representing the EM field are defined as usual in terms of the EM field tensor $F_{\mu\nu}$ and polarization tensor $P_{\mu\nu}$

$$E_\mu = F_{\mu\nu}\frac{V^\nu}{c}, \qquad B_\mu = F^*_{\mu\nu}\frac{V^\nu}{c} \;, \tag{9}$$

$$D_\mu = P_{\mu\nu}\frac{V^\nu}{c}, \qquad H_\mu = P^*_{\mu\nu}\frac{V^\nu}{c} \;, \tag{10}$$

where the dual tensor $X^*_{\mu\nu}$ is defined as $X^*_{\mu\nu} = \frac{1}{2}\eta_{\mu\nu\alpha\beta}X^{\alpha\beta}$, for any antisymmetric second-order tensor $X_{\alpha\beta}$. The meaning of the vectors D^μ and H^μ comes from the Lagrangean of the EM field, and in the case of a vacuum they are

$$H_\mu = \mu_{\mu\nu}B^\nu, \qquad D_\mu = \epsilon_{\mu\nu}E^\nu \;, \tag{11}$$

where the permittivity tensors are given as

$$\mu_{\mu\nu} = \left[1 + \frac{2\alpha}{45\pi B_q^2}\left(B^2 - E^2\right)\right] h_{\mu\nu} - \frac{7\alpha}{45\pi B_q^2}E_\mu E_\nu \;, \tag{12}$$

$$\epsilon_{\mu\nu} = \left[1 + \frac{2\alpha}{45\pi B_q^2}\left(B^2 - E^2\right)\right] h_{\mu\nu} + \frac{7\alpha}{45\pi B_q^2}B_\mu B_\nu \;. \tag{13}$$

In these expressions α is the EM coupling constant ($\alpha = \frac{e^2}{\hbar c} = \frac{1}{137}$) and B_q is a quantum electrodynamic parameter, $B_q = \frac{m^2c^3}{e\hbar} = 4.41 \times 10^{13}$ G, also known as the Schwinger critical B-field, i.e., $B_q \equiv B_{crit}$. The tensor $h_{\mu\nu}$ is the metric induced in the reference frame perpendicular to the observers, determined by the vector field V^μ. Our main concern in this paper is the behavior of NLED in either a pulsar or a magnetar, so in this particular case, $E^\alpha = 0$, $\epsilon^\alpha_\beta = \epsilon h^\alpha_\beta + \frac{7\alpha}{45\pi B_q^2}B^\alpha B_\beta$ and $\mu_{\alpha\beta} = \mu h_{\alpha\beta}$. The scalars ϵ and μ can be read directly from Eqs.(12,13) as $\epsilon \equiv \mu = 1 + \frac{2\alpha}{45\pi B_q^2}(B^2)$. We will deal with light propagation in NLED in optical approximation. The EM wave is represented by 3-surfaces of discontinuities of the EM field that propagate in the nonlinear background. As we show below, the EM wave propagation can be described as if the metric of the background were changed from its original form determined by GR into another effective metric that depends on the dynamics of the background EM field. This formalism allows us to use the well-known results from Riemann geometry largely applied in GR.

Following Hadamard (1903), the surface of discontinuity of the EM field is denoted by Σ. The field is continuous when crossing Σ, while its first derivative presents a finite discontinuity, specified as follows

$$[B^\mu]_\Sigma = 0, \quad [\partial_\alpha B^\mu]_\Sigma = b^\mu k_\alpha, \quad [\partial_\alpha E^\mu]_\Sigma = e^\mu k_\alpha \;, \tag{14}$$

where the symbol

$$[J]_\Sigma = \lim(J_{\Sigma+\delta} - J_{\Sigma-\delta}) \tag{15}$$

represents the discontinuity of the arbitrary function J through the surface Σ. The tensor $f_{\mu\nu}$ is called the discontinuity of the field, and $k_\lambda = \partial_\lambda\Sigma$ is the propagation vector. Applying conditions (14) and (15) to the field equations in the particular case of $E^\alpha = 0$, we obtain the constrains $e^\mu\epsilon_{\mu\nu}k^\nu = 0$ and $b^\mu k_\mu = 0$ and the following equations for the discontinuity fields e_α and b_α:

$$\epsilon^{\lambda\gamma}e_\gamma k_\alpha\frac{V^\alpha}{c} + \eta^{\lambda\mu\rho\nu}\frac{V_\rho}{c}\left(\mu b_\nu k_\mu - \mu' \lambda_\alpha B_\nu k_\mu\right) = 0 \;, \tag{16}$$

$$b^\lambda k_\alpha\frac{V^\alpha}{c} - \eta^{\lambda\mu\rho\nu}\frac{V_\rho}{c}\left(e_\nu k_\mu\right) = 0 \;. \tag{17}$$

Isolating the discontinuity field from (16), substituting in equation (17), and expressing the products of the completely anti-symmetric tensors $\eta_{\nu\xi\gamma\beta}\eta^{\lambda\alpha\rho\mu}$ in terms of delta functions, we obtain

$$\begin{aligned} &b^\lambda(k_\alpha k^\alpha)^2 + \left(\frac{\mu'}{\mu}l_\beta b^\beta k_\alpha B^\alpha + \frac{\beta B_\beta b^\beta B_\alpha k^\alpha}{\mu - \beta B^2}\right)k^\lambda \\ &+ \left(\frac{\mu'}{\mu l_\alpha b^\alpha}\left[(k_\beta V^\beta)^2(k_\alpha k^\alpha)^2\right] - \frac{\beta B_\alpha b^\alpha(k_\beta k^\beta)^2}{\mu - \beta B^2}\right)B^\lambda \\ &- \left(\frac{\mu'}{\mu}l_\mu b^\mu k_\alpha B^\alpha k_\beta V^\beta\right)V^\lambda = 0 \;. \end{aligned} \tag{18}$$

This expression is already squared in k_μ but still has an unknown b_α term. To get rid of it, one multiplies by B_λ, to

take advantage of the EM wave polarization dependence. By noting that if $B^\alpha b_\alpha = 0$ one obtains the *dispersion relation* by separating out the $k^\mu k^\nu$ term, what remains is the (-) effective metric. Similarly, if $B_\alpha b^\alpha \neq 0$, one simply divides by $B_\gamma b^\gamma$ so that by factoring $k^\mu k^\nu$, what results is the (+) effective metric. For the case $B_\alpha b^\alpha = 0$, one obtains

$$g^{\alpha\beta} k_\alpha k_\beta = 0 \; . \tag{19}$$

whereas for the case $B_\alpha b^\alpha \neq 0$, the result is

$$\left[\left(1 + \frac{\mu' B}{\mu} + \frac{\beta B^2}{\mu - \beta B^2}\right) g^{\alpha\beta} - \frac{\mu' B}{\mu}\frac{V^\alpha V^\beta}{c^2} + \left(\frac{\mu' B}{\mu} + \right.\right.$$
$$\left.\left. + \frac{\beta B^2}{\mu - \beta B^2}\right) l^\alpha l^\beta\right] k_\alpha k_\beta = 0 \; , \tag{20}$$

where by $(')$ we mean $\frac{d}{dB}$, and we define $\beta = \frac{7\alpha}{45\pi B_q^2}$, and $l^\mu \equiv \frac{B^\mu}{|B^\gamma B_\gamma|^{1/2}}$.

From the above expressions we can read the effective metric $g_+^{\alpha\beta}$ and $g_-^{\alpha\beta}$, where the labels "+" and "-" refers to extraordinary and ordinary polarized rays, respectively. To determine the redshift we need the covariant form of the metric tensor, obtained from the expression $g_{\mu\nu} g^{\nu\alpha} = \delta^\alpha{}_\mu$. It reads

$$g^-_{\mu\nu} = g_{\mu\nu} \tag{21}$$

and

$$\begin{aligned} g^+_{\mu\nu} &= \left(1 + \frac{\mu' B}{\mu} + \frac{\beta B^2}{\mu - \beta B^2}\right)^{-1} g_{\mu\nu} \\ &+ \left[\frac{\mu' B}{\mu(1 + \frac{\mu' B}{\mu} + \frac{\beta B^2}{\mu - \beta B^2})(1 + \frac{\beta B^2}{\mu - \beta B^2})}\right] \frac{V_\mu V_\nu}{c^2} \\ &+ \left(\frac{\frac{\mu' B}{\mu} + \frac{\beta B^2}{\mu - \beta B^2}}{1 + \frac{\mu' B}{\mu} + \frac{\beta B^2}{\mu - \beta B^2}}\right) l_\mu l_\nu \; . \end{aligned} \tag{22}$$

The function $\frac{\mu' B}{\mu}$ can be expressed in terms of the magnetic permissivity of the vacuum, and is given as

$$\frac{\mu' B}{\mu} = 2\left(1 - \frac{1}{\mu}\right) \; . \tag{23}$$

In the particular case that we are focusing on, both the emitter and observer are in inertial frames, that is, $V^\mu = \delta^\mu_0/(g_{00})^{1/2}$; therefore, the components of both effective metrics above become coincident and given as

$$g_{00}^{\rm eff} = \frac{g_{00}}{1 + \frac{\beta B^2}{\mu - \beta B^2}} \; . \tag{24}$$

The general expression for the redshift is then given as

$$\frac{\nu_B}{\nu_A} = \frac{\lambda_A}{\lambda_B} = \left[\frac{g_{00}(e)}{g_{00}(o)}\right]^{\frac{1}{2}} \; , \tag{25}$$

or

$$z = \frac{\lambda_B - \lambda_A}{\lambda_A} = \left[\frac{g_{00}(e)}{g_{00}(o)}\right]^{-\frac{1}{2}} - 1. \tag{26}$$

where $g_{00}(e)$ and $g_{00}(o)$ stand for the time-time effective metric components at emission and observation, respectively. Hence, for observations very far from the star the redshift can be approximated as

$$z + 1 = \left[\frac{g_{00}(o)}{g_{00}(e)}\right]^{\frac{1}{2}} = \frac{\left(1 - \frac{2GM}{c^2 R}\right)^{-\frac{1}{2}}}{\left[1 + \frac{\beta B^2}{\mu - \beta B^2}\right]^{\frac{1}{2}}} \; . \tag{27}$$

$$z + 1 \simeq \frac{\left(1 - \frac{2GM}{c^2 R}\right)^{-\frac{1}{2}}}{[1 + \beta B^2]^{\frac{1}{2}}} = \frac{\left(1 - 0.3\frac{M}{R}\right)^{-\frac{1}{2}}}{[1 + 0.19 B_{15}^2]^{\frac{1}{2}}} \tag{28}$$

where M is the star mass in units of $M_\odot$, R its radius in units of 10 km, and B_{15} is the B-field in units of 10^{15} G.

Note, however, that in the present case the correction on the gravitational redshift brought to z by the nonlinear contribution of the magnetic field does depend on the polarization b^μ of the emitted photons. Therefore, it is straightforward to verify that because of the appearance of the two different effective metrics in equations (22,21), which exihibit the phenomenon of birefringence, one may *in principle* disentangle the two components of the total pulsar surface redshift by a direct observation.

4. DISCUSSION AND CONCLUSION

The 5.0 keV feature discovered with the *Rossi X-ray Timing Explorer* is strong, with an equivalent width of ~ 500 eV and a narrow width of less than 0.4 eV (Ibrahim et al. 2002, 2003). When these features are viewed in the context of accretion models, one arrives to a $M/R > 0.3$ $M_\odot$ km^{-1}, which is inconsistent with NSs, or requires a low $B \sim (5 - 7) \times 10^{11}$ G, which is said not to correspond to any SGRs (Ibrahim et al. 2003). In the magnetar scenario, meanwhile, the features are plausibly explained as being ion-cyclotron resonances in an ultra strong B-field, $B_{\rm sc} \sim 10^{15}$ G, whose energy and width are close to model predictions (Ibrahim et al. 2003). According to Ibrahim et al. (2003), the confirmation of this findings would allow to estimate the gravitational redshift, mass, and radius of the supposed magnetar SGR 1806-20.

Here we point out the possibility that this feature could also be due to NLED in the same superstrong B-field of SGR 1806-20, as suggested by equation (27). To obtain our conclusion, we used $B \sim 5 \times 10^{15}$ G, which is within the uncertainty of the B-field strength estimate from P and $\dot{P}$ and the likely B-field near-surface multipole structure, as suggested by various authors in the field. In particular, Duncan (1998) interpreted the 23 ms *global* oscillations observed in the "magnetar-like" object SGR 0526-66 as being a fundamental *toroidal mode*, assuming a field $B \sim 4 \times 10^{15}$ G lies underneath the star crust. Other authors hintat the coexistence of *poloidal* configurations as well. For such fields, the cyclotron viewpoint could be sustained only whenever the dipole component is the dominant emission mechanism. If this were the case, no conclusive assertion about the M/R ratio of the compact star glowing in SGR 1806-20 could be consistently made, since the NLEDredshift might well be mimicking

the standard gravitational redshift associated with the pulsar surface. More fundamentally yet, if new spectral lines were measured with high precision (as in Cottam et al. 2002) from heavy elements in a compact object with fields $B \gtrsim 10^{15}$ G, then the $\Delta z \gtrsim 10$ z-correction brought by NLED would prove critical regarding both its M/R ratio and its EoS.

As a worthy remark, the attentive reader must realize that there exists a hidden divergence in the effective metric here derived. It appears when the magnetic field strength achieves values around $B \longrightarrow B_q \sim 10^{13.5}$ G. We stress that such a divergence is inherent to the sort of approximation we are using for, that is, the Heisenberg & Euler (1936) Lagrangean, which is not an exact one, of which we just take into account only the first term in its expansion. We advance, meanwhile, that such divergence can be removed by taken advantage of a very different sort of nonlinear electrodynamics Lagrangean, as the exact one introduced by Born & Infeld (1934). This new approach is matter of a forthcoming communication.

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